OPTIMAL TAXATION OF FOSSIL FUEL IN GENERAL EQUILIBRIUM

Mistra 2010

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- But – the effects of global warming and of policy are;
  - *i.* uncertain, *ii.* long-run and dynamic, *iii.* have potentially large general equilibrium effects, and *iv.* may crucially depend on technological change.
- Show that reasonable simplifications make it possible to do simple, yet reasonable back-of-the-envelope calculations of the size of optimal carbon taxes under uncertainty.

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  3. An *economic model of the world* where the global mean temperature causes damages to GDP and emissions come from economic activity.

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3. Consider backstop technology and discuss other extensions.
4. Concluding thoughts.

The base line model (yet no uncertainty and no backstop)

- Planning problem (Dasgupta & Heal plus externality);

$$\max_{\{C_t, K_{t+1}, E_t, R_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$\begin{aligned} C_t + K_{t+1} &= D(S_t; \gamma_s) F(A_t, K_t, E_t, A_t^e) + (1-\delta) K_t \\ R_{t+1} &= R_t - E_t, \quad R_0 \text{ given}, \\ R_t &\geq 0 \forall t, \\ S_t &= L(E^t). \end{aligned}$$

where K_t is capital, E_t is fossil fuel use, R_t is remaining fossil fuel in ground, S_t is the stock of carbon in the atmosphere (in the form of CO_2), A_t, A_t^e are technology trends in TFP and energy augmentation.

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- γ_S parametrizes the damage function D, later to be specified. I will later allow uncertainty about it.

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- Thus, we need to *price the externality.*
- We want to find how much a marginal unit of CO_2 emission dynamically reduces welfare and express this in consumption units (dollars).
- Turn out that we a few fairly reasonable simplifications, we can do this very easily.

The marginal price of the externality

- We can show that the price of the externality satisfies

$$\Lambda_t^s \equiv -\sum_{s=1}^{\infty} (1-\varphi)^{s-1} \beta^j \frac{U'(C_{t+s})}{U'(C_t)} \left(\frac{D'(S_{t+s})}{D(S_{t+s})} Y_{t+s} \right).$$

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- Λ_t^s measures the total (dynamic) cost of a marginal unit of carbon in the atmosphere in terms of the consumption good.

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- Here, these exactly cancel.

Simplifications

- Recall that dynamic *expected* marginal climate damages are given by

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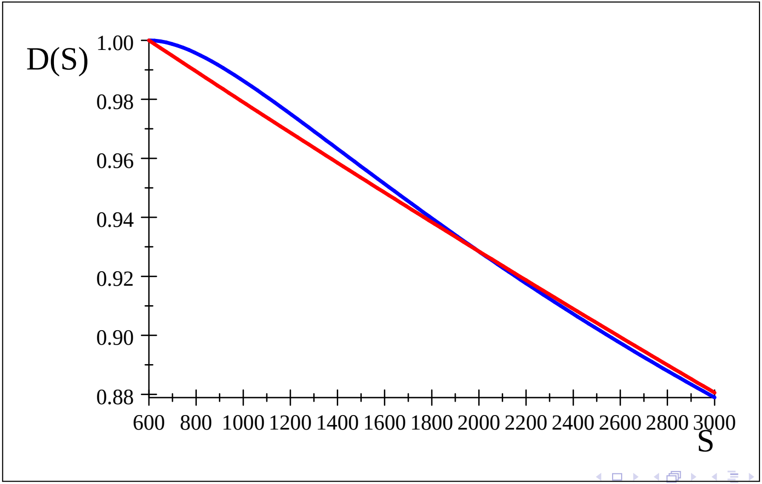
- Note that future output (driven by productivity, population or what ever) is not part of Λ_t^s. Damages and thus optimal taxes are driven by future stocks of carbon in atmosphere and proportional to current output. Makes analysis very simple.

Simplifications cont'd.

- Let's assume $S(S_t; \gamma_{S,t}) = e^{-\gamma_{S,t} S_t}$ allowing stochastic variation in $\gamma_{S.t}$. Not terribly different from state-of-the-art damage function (Nordhaus, DICE 2007, in blue).



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- It also implies that optimal law-of-motion for E_t is completely determined by Γ_t^e. The rest of the system very easy to solve.

Stochastic damages - a calibration example

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- $\Lambda_t^s = Y_t \frac{\beta}{1-\beta(1-\varphi)} \bar{\gamma}_S$.

Stochastic damages - a calibration example

- Consider the case when $\gamma_{S,t}$ follows a simple stochastic process. γ_S starts at an initial value and at some possibly random time, it changes to $\gamma_S \in \left\{\gamma^L, \gamma^H\right\}$. Conditional on a change, $\gamma_s = \gamma^H$ happens with probability p. Can be interpreted as learning.
- Until we learn, $\gamma_{S,t} = (1-p)\,\gamma^L + p\gamma^H \equiv \bar{\gamma}_S$ for simplicity (but not necessity).
- Optimal tax is now very easy to find.
- Before learning;
- $\Lambda_t^s = Y_t \frac{\beta}{1-\beta(1-\varphi)} \bar{\gamma}_S$.
- After it is $Y_t \frac{\beta}{1-\beta(1-\varphi)} \gamma^H$ or $Y_t \frac{\beta}{1-\beta(1-\varphi)} \gamma^L$.

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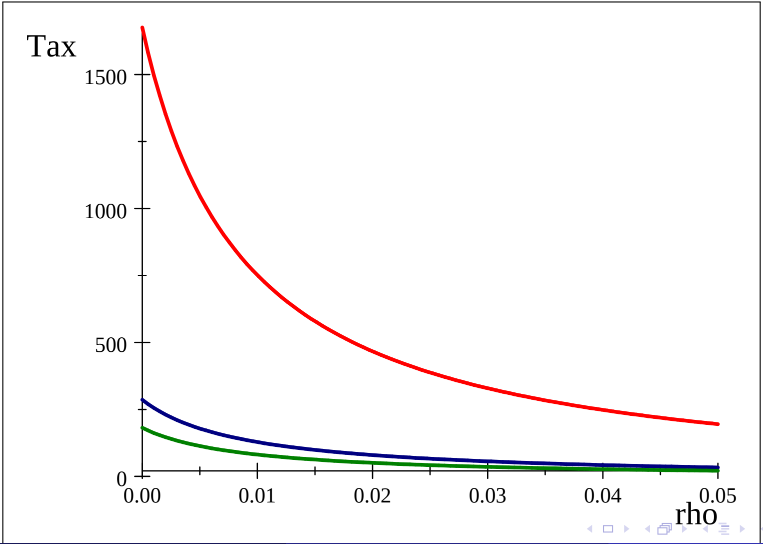
- Set global yearly output Y to $70*10^{12}$ US dollar.

Results

Optimal current tax per ton carbon (at current output) vs. rate of pure time preference



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- Optimal tax not very affected by low probability catastrophic event (*with log utility*).
- Investment in alternative technologies may be more sensitive to such an event (*insurance value*).
- Taxes should now be introduced abruptly. Due to Cobb-Douglas assumption, which is clearly unrealistic in very short run. However, due to targeted technical change perhaps not a bad assumption in medium run.

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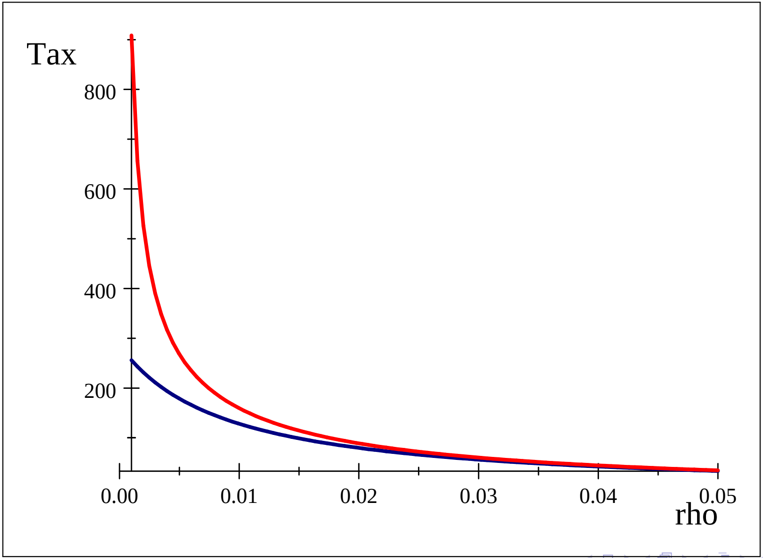
- $Y_t \left(\frac{(1-\varphi_0)\beta}{1-\beta(1-\varphi)} + \frac{\varphi_0 \gamma \beta}{1-\beta} \right)$
- This makes a lot of difference if β is close to one. Logical!

Results

Optimal current tax per ton carbon. (red $\varphi_0 = 0.3$,blue $\varphi_0 = 0$)



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- With 30% carbon remaining forever, the figures are 909 vs 114 $/ton.
- But, neither of them have well developed GE model. Hard to interpret their results.

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- In the decentralized allocation, optimal taxes will then drive prices equal to extraction costs – no rents.

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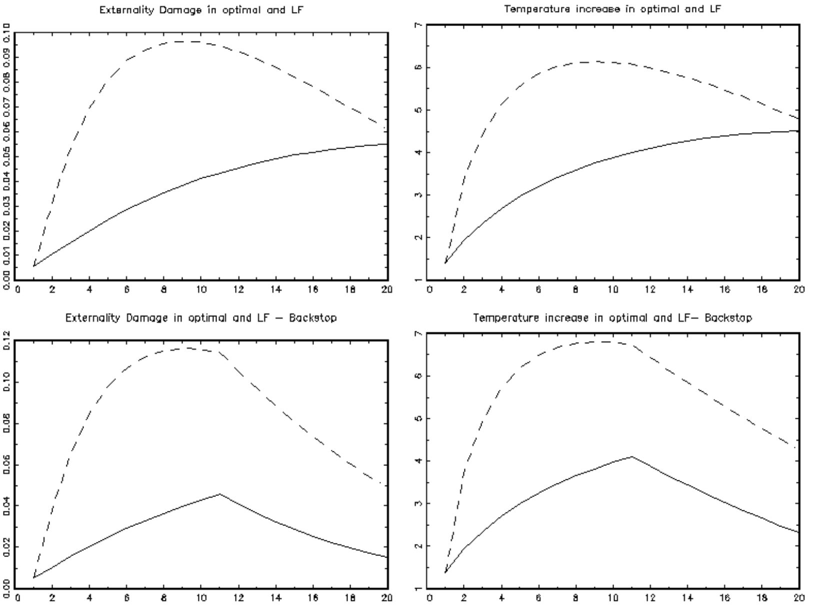
- Assume, for example, backstop makes fossil fuel for sure obsolete in 100 years. Optimal total use of 870 GtC is much less than what we know exists.
- Optimal tax policy does not depend on backstop – but welfare loss of not implementing it depends crucially on when backstop arrives since in *LF* all fossil fuel will be extracted.

Comparison between optimal policy and Laissez-faire



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- When backstop technology arrives is uncertain. But;
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- we are sure it arrives before we run out of fossil fuel.

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- Easy to derive optimal policy given current estimates of damages.
- Back-of-the-envelope calculations of optimal policy seems fairly robust, BUT
- we need much more understanding about damages, adaptation and technical change as well as regarding coal cycle persistence.